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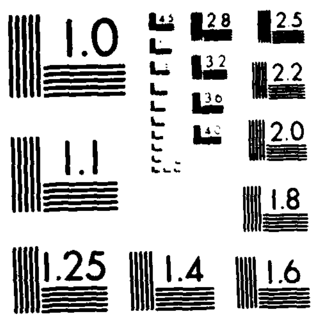

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A HYBRID FEM-T-MATRIX TECHNIQUE FOR THE ANALYSIS

OF ACOUSTIC WAVE SCATTERING BY ELASTIC SHELLS OF REVOLUTION

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ABSTRACT

A hybrid finite element cum T-matrix technique is introduced for the analysis of problems involving acoustic wave scattering by shells of revolution immersed in water. The concept of a mechanical impedance or receptance is used from the theory of vibrations to provide a relationship between the velocity of the shell surface and the pressure acting upon it. This relationship is introduced in the Helmholtz integral relations for the incident and scattered fields and a T-matrix for the scattering problem is derived. The mechanical impedance is calculated by a conventional FEM technique. Numerical results are obtained for spherical shells as a check on the numerical procedure and also for a finite capped cylindrical shell for waves incident along the rotational axes of symmetry. The comparison with the experimental results of Dr. S. Numrich and Dr. L. Dragonette of the Naval Research Laboratory is excellent.

INTRODUCTION

In this report a hybrid method employing the T-matrix approach[1,2] along with a finite element formulation is introduced for the analysis of problems involving the scattering of sound waves by elastic shells immersed in water. It is shown that by using the concept of a mechanical impedance, we can avoid the use of integral representations in the interior of the scatterer or the use of fluid elements in the exterior and 3-D elements in the interior of the shell. In other words, this method exploits the best features of both the T-matrix method and the FEM. The latter of course can successfully handle several geometries of practical interest.

The impedance of the shell is calculated and it provides the necessary relationship between the velocity on the shell surface and the pressure acting on it. Thus the number of unknowns in the scattering problem is reduced by one. The integral representation of the scattered field and the extincting theorem are then used in the conventional manner to yield a T-matrix for the problem. Another byproduct of the receptance calculation is that the free vibration frequencies of the shell are also generated many of which can be identified with the maxima and minima of the spectrum of the scattered pressure field.

Previous calculations for acoustic wave scattering from elastic shells of revolution[1] experienced numerical difficulties as the shell thickness decreased and the frequency increased. Moreover the formulation used invoked integral representations in the interior of the shell region, this resulted in very complicated expressions for the T-matrix which depended on 5 or 6 different matrices. Some of these matrices are quite ill conditioned. With the present scheme, the scattering problem is a purely scalar one and hence much better conditioned numerically.

FORMULATION OF THE SCATTERING PROBLEM

Consider a shell of revolution bounded by the surface S with a piecewise continuous outward normal $\hat{n}$ that is immersed in water. The acoustic field in water is described by the scalar potential ϕ. We consider a time harmonic plane wave of frequency ω incident on the shell. Without loss of generality, the plane of incidence can be considered to be the x-z plane. Further the time dependence of all fields is $\exp(-i\omega t)$ and this is suppressed for brevity. The total field outside the shell is given by

$$\phi(\vec{r}) = \phi_o(\vec{r}) + \phi_s(\vec{r}) \ ; \ \vec{r} \in V \tag{1}$$

where ϕ_o and ϕ_s are the incident and scattered fields respectively.

The free space Green's function in the fluid is given by

$$g(\vec{r},\vec{r}\,') = \exp ik_f |\vec{r}-\vec{r}\,'| / |\vec{r}-\vec{r}\,'| \tag{2}$$

where $k_f = \omega/c_f$ is the wavenumber in the fluid, c_f is the sound speed in the fluid and $\vec{r}$ and $\vec{r}\,'$ are respectively the field and source points.

The scalar potential satisfies a homogeneous wave equation in V and so also does g except at $\vec{r} = \vec{r}\,'$. The integral representation for such a system is known to be

$$\frac{1}{4\pi} \int_S \{\phi_+' \hat{n}' \cdot \nabla' g(\vec{r},\vec{r}\,') - g(\vec{r},\vec{r}\,')\hat{n}' \cdot \nabla_+' \phi\} dS' = \begin{cases} -\phi_o(\vec{r}) & ; \ \vec{r} \notin V \\ \phi_s(\vec{r}) & ; \ \vec{r} \in V \end{cases} \tag{3}$$

The first form of Eq. (3) for $\vec{r} \notin V$, is known as the extinction theorem and the second form of Eq. (3) for $\vec{r} \in V$ is the integral representation of the scattered field. The surface quantity $\phi_+ = (i\omega\rho_f)^{-1} p_+$, where p_+ is the surface pressure and ρ_f is the mass density of the fluid. The surface quantity $\hat{n} \cdot \nabla_+ \phi = \hat{n} \cdot \vec{V}_+$ where $\vec{V}_+$ is the fluid particle velocity on S.

Both these quantities are unknown.

In the T-matrix approach, ϕ_o, ϕ_s and g are all expanded in a set of spherical functions that are given by

$$\begin{matrix}\mathrm{Ou}\\ \mathrm{Re}\end{matrix}\psi_{\ell m\sigma}(\vec{r}) = \left\{\begin{matrix} h_\ell^{(1)}(k_f r) \\ j_\ell(k_f r)\end{matrix}\right\} Y_{\ell m\sigma}(\theta,\phi) \tag{4}$$

where $h_\ell^{(1)}$ are spherical Hankel functions of the first kind, j_ℓ are the spherical Bessel functions and $Y_{\ell m\sigma}$ are normalized spherical harmonics. The index $\ell \in [0,\infty]$, $m \in [0,\ell]$ and σ denotes the angular parity - even or odd. The incident plane wave

$$\phi_o(\vec{r}) = e^{ik_f\hat{k}_o\cdot\vec{r}} = \sum_{\ell m\sigma} a_{\ell m\sigma}(\hat{k}_o)\mathrm{Re}\phi_{\ell m\sigma}(\vec{r}) \tag{5}$$

where

$$a_{\ell m\sigma} = 4\pi i^{\ell} Y_{\ell m\sigma}(\hat{k}_o) \tag{6}$$

and $\hat{k}_o$ is the direction of propagation of the incident wave.

The scattered field satisfies radiation conditions at infinity and hence is expanded in outgoing functions

$$\phi_s(\vec{r}) = \sum_{\ell m\sigma} f_{\ell m\sigma}\,\mathrm{Ou}\psi_{\ell m\sigma}(\vec{r}) \tag{7}$$

where the expansion coefficients $f_{\ell m\sigma}$ are to be determined.

The expansion of g is known to be

$$g(\vec{r},\vec{r}\,') = ik_f \sum_{\ell m\sigma} \left\{\begin{matrix} \mathrm{Ou}\psi_{\ell m\sigma}(\vec{r})\mathrm{Re}\psi_{\ell m\sigma}(\vec{r}\,')_j & |\vec{r}|>|\vec{r}\,'| \\ \mathrm{Ou}\psi_{\ell m\sigma}(\vec{r}\,')\mathrm{Re}\psi_{\ell m\sigma}(\vec{r})_j & |\vec{r}\,'|>|\vec{r}| \end{matrix}\right. \tag{8}$$

Substituting Eqs. (5), (7) and (8) in the two forms of Eq. (3) and using the orthogonality of the angular parts of the spherical basis functions we have

$$-a_{\ell m\sigma} = \frac{ik_f}{4\pi}\int \{\ \phi_+\hat{n}\cdot\nabla\mathrm{Ou}\psi_{\ell m\sigma} - \mathrm{Ou}\psi_{\ell m\sigma}\hat{n}\cdot\nabla_+\phi\}\ dS \tag{9}$$

and

$$f_{\ell m\sigma} = \frac{ik_f}{4\pi} \int \{\phi_+ \hat{n}\cdot\nabla \mathrm{Re}\psi_{\ell m\sigma} - \mathrm{Re}\psi_{\ell m\sigma}\hat{n}\cdot\nabla_+\phi\} dS \tag{10}$$

The surface pressure and normal component of the particle velocity are related through the mechanical receptance of the shell as follows.

We expand the normal component of the velocity on the shell surface

$$\hat{n} \cdot \nabla_+\phi = \sum_n C_n\, q_n \tag{11}$$

where q_n are any set of basis functions associated with the shell. In the next section, we shall identify q_n with the eigenvectors of the shell. The pressure on the surface of the shell is expanded as shown below

$$\phi_+ = \sum_n B_n\, q_n \tag{12}$$

The mechanical receptance of the shell is the ratio of the surface velocity to the surface pressure and in the next section we show that

$$C_n = \sum_{n'} R_{nn'}\, B_{n'} \tag{13}$$

where R is the receptance matrix. Substituting Eqs. (11)-(13) in (9) and (10) we have

$$a_n = -i \sum_{n'} Q_{nn'}(\mathrm{Ou}\psi_n, q_{n'}) B_{n'} \tag{14}$$

$$f_n = i \sum_{n'} Q_{nn'}(\mathrm{Re}\psi_n, q_{n'}) B_{n'} \tag{15}$$

where the superindex $n \to (\ell, m, \sigma)$ and

$$Q_{nn'}(\mathrm{Ou}\psi_{\ell m\sigma}, q_{n'}) = \frac{k_f}{4\pi} \sum_{n''} \int_S \{(\hat{n}\cdot\nabla^{\mathrm{Ou}}_{\mathrm{Re}}\, \psi_{\ell m\sigma})$$

$$\delta_{n'n''} - R_{n'n''}{}^{\mathrm{Ou}}_{\mathrm{Re}}\psi_{\ell m\sigma} \} q_{n''} dS \tag{16}$$

Equations (14) and (15) can now be formally solved to yield

$$f_n = \sum_{n'} T_{nn'} a_{n'} \tag{17}$$

where

$$T = - Q(Re\psi,q)[Q(Ou\psi,q)]^{-1} \tag{18}$$

is the T-matrix of the scatterer.

The following properties of the T-matrix resulting from reciprocity and the conservation of energy can be shown

$$T = T^t \quad \text{(symmetry)} \tag{19}$$

$$TT^* = - \mathrm{Real}(T) \tag{20}$$

The above properties can be used as a check of the numerical computations.

CALCULATION OF MECHANICAL RECEPTANCE

To calculate the receptance the shell theory of elasticity is used in conjunction with a finite element approximation. Consider a shell of revolution of thickness 2h. The variable s is a length coordinate measured on the mid-surface of the shell (see Fig. 1). Let u and w be the normal and tangential displacements of the shell surface and β is a meridional deflection. According to the assumptions of shell theory[2,3], the strains and displacements are related as follows

$$\begin{bmatrix} \varepsilon_s \\ \varepsilon_\theta \\ \chi_s \\ \chi_\theta \\ \gamma \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial s} + \frac{w}{R} \\ (w\cos\phi + u\sin\phi)/r \\ -\frac{d\beta}{ds} + \frac{d}{ds}(u/R) \\ -\sin\phi(\beta - u/R)/r \\ \frac{dw}{ds} - \beta \end{bmatrix} \tag{21}$$

where R is the principal radius of curvature in the u-w plane and r and ϕ are measured as shown in Fig. 1. In Eq. (21), ε_s and ε_θ are membrane strains, χ_s and χ_θ are bending strains and γ is a transverse shear strain.

The strain energy of the shell can be written as

$$\Pi = \frac{1}{2} \int \varepsilon^T D \varepsilon \, dS - \int f^T u \, dS \tag{22}$$

where

$$\vec{u} = \begin{Bmatrix} u \\ w \\ \beta \end{Bmatrix} \tag{23}$$

is the displacement vector and $\vec{f}$ is the vector of the external force and D is the elastic matrix

$$D = \begin{bmatrix} D_1 & & \\ & D_2 & \\ & & D_3 \end{bmatrix} \tag{24}$$

where for linear, isotropic materials

$$D_1 = \frac{Eh}{1-\nu^2}\begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} \; ; \quad D_2 = \frac{Eh^3}{12(1-\nu^2)}\begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} \; ; \quad D_3 = \frac{5}{12}\,\frac{Eh}{1+\nu} \tag{25}$$

In Eq. (25), E is the Young's modulus and ν is the Poisson's ratio.

Through the usual finite element approximation process[4], the displacements at any point on the surface of the shell are written as

$$\vec{U}(\vec{r}) = N_i\vec{U}_i + N_{i+1}\,\vec{U}_{i+1} \quad ; \quad \vec{r} \in E_i \tag{26}$$

where N_i are shape functions and i refers to the i-th node and E_i is the i-th element. U_i and U_{i+1} are the nodal values at the edges of the element E_i. We note that for axisymmetric problems, there is no dependence of the fields on the azimuthal angle.

Using Eq. (26) in (21) and (22), the element stiffness matrix can be written as

$$K = \int_{-1}^{+1} B_i^T D B_{i+1} dS \tag{27}$$

where

$$N_i = \frac{1-\eta}{2} \text{ and } N_{i+1} = \frac{1+\eta}{2}$$

where 'η' is measured along the curved length of the element.

The matrix B is given by

$$B_i = \begin{bmatrix} \frac{\partial N_i}{\partial s} & \frac{N_i}{R} & 0 \\ \frac{N_i \sin\phi}{r} & \frac{N_i \cos\phi}{r} & 0 \\ \frac{1}{R}\frac{dN_i}{ds} & 0 & -\frac{dN_i}{ds} \\ \frac{N_i \sin\phi}{rR} & 0 & -\frac{N_i \sin\phi}{r} \\ 0 & \frac{dN_i}{ds} & -Ni \end{bmatrix} \tag{28}$$

The element mass matrix can be derived from the expression for the kinetic energy which after integration in the thickness direction and the azimuthal direction can be written as

$$T = \frac{1}{2}(\rho h) \int [\dot{u}^2 + \dot{w}^2 + \frac{h^2}{12} \beta^2\phi]2\pi r ds \tag{29}$$

where ρ is the mass density of the shell. The last term in the integrand of Eq. (29) is the rotary inertia term.

The minimization of the Lagrangian leads to the following matrix equation

$$K\{Q\} + M\{\ddot{Q}\} = 0 \tag{30}$$

where {Q} is the nodal vector of modal values of the displacement and M is the mass matrix that results when Eq. (26) is substituted in (29) and the Lagrangian is minimized. If one assumes that the time dependence of {Q} is of the form

$$\{Q\} = q_n e^{i\omega_n t} \tag{31}$$

then

$$K\{q_n\} = -\omega_n^{\,2} M\{q_n\} \tag{32}$$

where ω_n is the n-th eigenvalue and $\{q_n\}$ is the corresponding mode shape. Since the eigenvectors are arbitrary up to a multiplicative factor, it is convenient to make use of the fact that $\{q_n\}$ are orthogonal with respect to the mass matrix and normalize the $\{q_n\}$ such that

$$\{q_n\}^T M\{q_{n'}\} = \delta_{nn'} \tag{33}$$

Once the eigenvalues and eigenvectors are determined for the free vibration of the shell, now the forced vibration problem is considered. The boundary condition on the shell-fluid interface is that the pressure and the normal component of the velocity are continuous. The surface value of the pressure is related to ϕ_+ and the equivalent nodal forces

can be written as

$$\{F\} = \{f\}e^{-i\omega t} = \sum_n B_n\{f_n\}e^{-i\omega t} \tag{34}$$

where the $\{f_n\}$ are derived from Eq. (12). The vibration problem now is

$$K\{Q\} + M\{\ddot{Q}\} = \{F\} \tag{35}$$

The nodal displacements are expanded in terms of the eigenmodes as

$$\{Q\} = \sum_n C_n\{q_n\} \tag{36}$$

Substituting Eqs. (34) and (36) in (35), we obtain

$$R_{nn'} = \frac{\{q_{n'}\}^T\{f_n\}}{\omega_n^2 - \omega^2} \qquad (3$$

This is the form of R used in the definition of the Q-matrix in Eq. (16)

RESULTS AND DISCUSSION

The first check of this method was done by performing calculations for a spherical shell. The top hemisphere of the shell was successively divided into 10 and then 20 elements. Both symmetric and anti-symmetric boundary conditions were applied at the equator and the eigenvalues and eigenvectors were generated by the usual finite element procedure. It was judged that 20 elements were sufficient to assure convergence. The eigenmodes of the sphere fall into two broad classes, the so-called bending and membrane nodes. In Table I, the natural frequencies for the spherical shell computed with 10 and 20 elements are compared with analytical results.

For the spherical capped cylinder where length (2ℓ) to diameter ($2a$) ratio is 1.6, (see Fig. 2), 35 elements were used for the upper half of the shell. In Table II the natural frequencies of the capped cylinder are tabulated. For the scattering problem a frequency range $0<\frac{\omega\ell}{2c_f}<15.0$ was considered and 44 eigenmodes were used in the calculation of the receptance. Out of these, 25 can be identified to be predominantly bending modes. The symmetry of the T-matrix was better than 1/2% in the entire frequency range $0 < k_f\ell/2 < 15.0$. Only the case of waves incident along the rotational axes of symmetry was considered. The frequency response of the scattered pressure field at distances far from the shell are plotted in Figs. 3, 4 and 5 for 3 different angles of observation. In Fig. 3, a portion of the calculated back scattered spectrum is compared with the experimental results provided to us by Dr. S. Numrich and Dr. L. Dragonette of the Naval Research Laboratory, Washington, D.C. The agreement is excellent. The experiment, which was of the pulse-echo type had a sampling interval of $\Delta k_f a = 0.1$ and hence the sharp resonance at $k_f a = 9.3$ which is seen in the calculation was missed while performing an FFT from the time to the

frequency domain. The experiment is very well calibrated and the maximum difference in the amplitude is less than 3db.

In conclusion, we have demonstrated a practical technique for the study of acoustic wave scattering by thin elastic shells of revolution. Due to the use of the finite element method for the calculation of the shell receptance, practical shapes and features such as stiffeners can be taken into consideration. The results from this new technique are far superior to that resulting from the use of a full elasticity solution for the shell. The approach is better conditioned numerically and is expected to yield stable results for aspect ratios greater than that by the conventional solution. It now remains to extend the calculation to oblique angles of incidence with respect to the rotational axes of symmetry.

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TABLE I

Natural Frequencies Ω of Spherical Shell in Nondimensional Units $\Omega = \sqrt{\ell R^2 \omega^2 / E}$

	10 elements	20 elements	Analytical
b_1	0.2905×10^{-3}	0.6774×10^{-4}	0.0
b_2	0.7221	0.7245	0.7259
b_3	0.8756	0.8814	0.8855
b_4	0.9786	0.9858	0.9939
b_5	1.107	1.108	1.212
b_6	1.295	1.274	1.296
b_7	1.561	1.495	1.528
b_8	1.915	1.771	1.818
b_9	2.364	2.100	2.167
b_{10}	2.914	2.479	2.570
b_{11}	3.574	2.904	3.026
b_{12}	4.351	3.374	3.532
a_0	1.78713	1.78713	1.78713
a_1	2.187	2.188	2.189
a_2	2.990	2.988	2.989
a_3	3.981	3.966	3.963
a_4	5.056	5.004	4.993
a_5	6.158	6.070	6.044
a_6	7.312	7.154	7.105
a_7	8.510	8.253	8.172
a_8	9.757	9.367	9.242
a_9		10.50	10.31
a_{10}		11.64	11.39
a_{11}		12.80	12.46
a_{12}		13.98	13.54

TABLE II

Natural Frequencies Ω of Finite Capped Cylinder, $\ell/2a=1.6$ in Non-dimensional Units ($\Omega = \sqrt{\ell R^2 \omega^2/E}$)

Bending Modes		Membrane modes	
b_1	= 0.0	a_o	5.387
b_2	= 2.639	a_1	7.522
b_3	= 3.213	a_2	8.904
b_4	= 3.696	a_3	11.45
b_5	= 3.854	a_4	14.33
b_6	= 4.038	a_5	17.08
b_7	= 4.331	a_6	20.13
b_8	= 4.731	a_7	23.11
b_9	= 5.207	a_8	26.09
b_{10}	= 5.893	a_9	32.23
b_{11}	= 6.53	a_{10}	35.31
b_{12}	= 7.343	a_{11}	41.55
b_{13}	= 8.257	a_{12}	44.7
b_{14}	= 9.243		
b_{15}	= 10.29		
b_{16}	= 11.48		
b_{17}	= 12.71		
b_{18}	= 14.01		
b_{19}	= 15.4		
b_{20}	= 16.84		
b_{21}	= 18.36		
b_{22}	= 19.8		
b_{23}	= 21.55		
b_{24}	= 22.24		

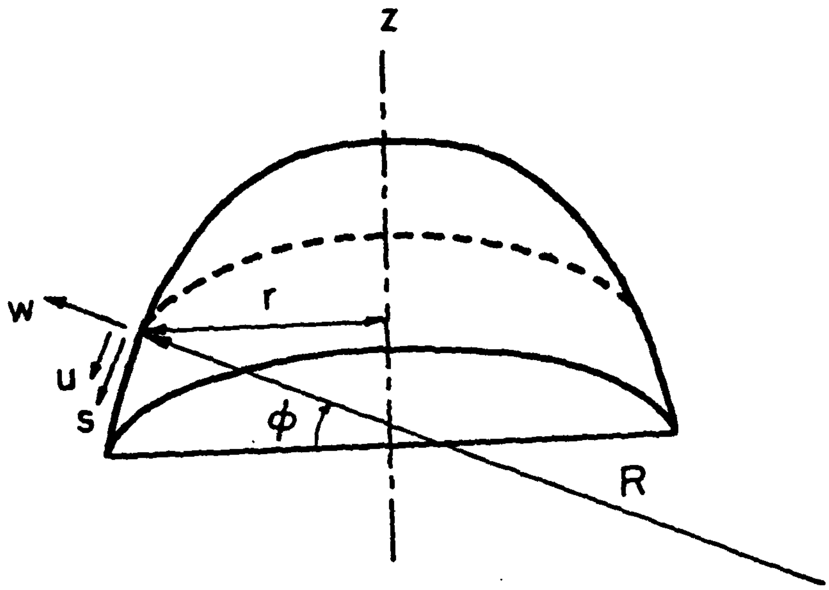

Fig. 1. Shell coordinates and displacements.

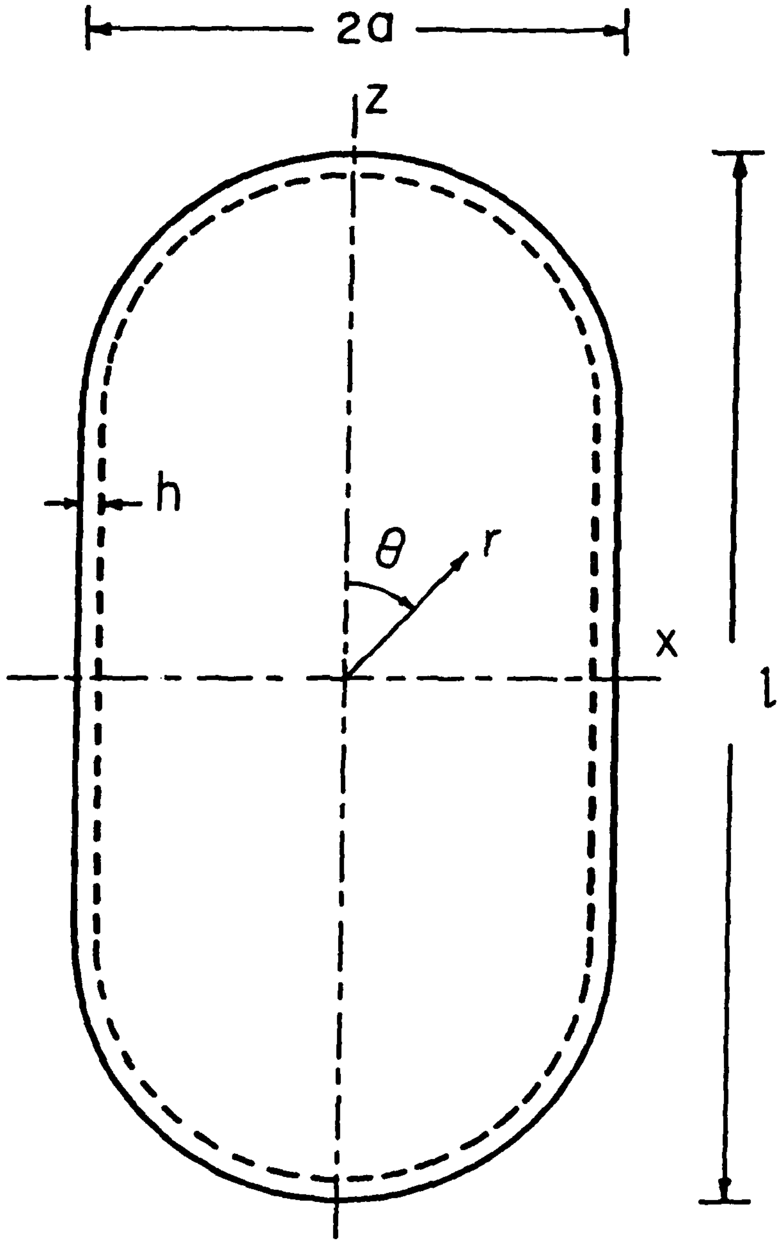

Fig. 2. Finite cylinder with spherical end caps.

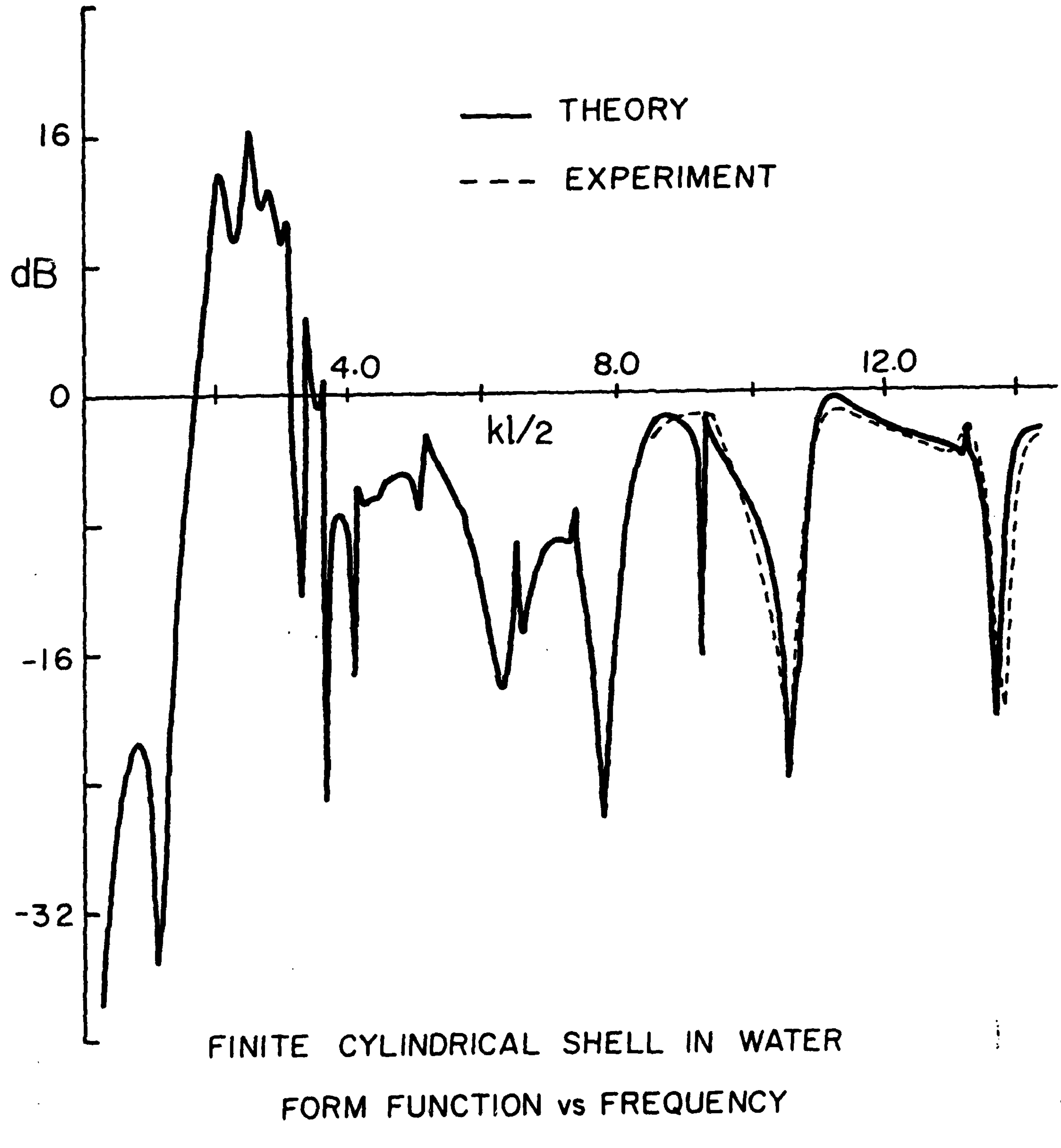

Fig. 3. Backscattered form function as a function of nondimensional wavenumber for normal incidence (θ_o=0, θ=180^o).

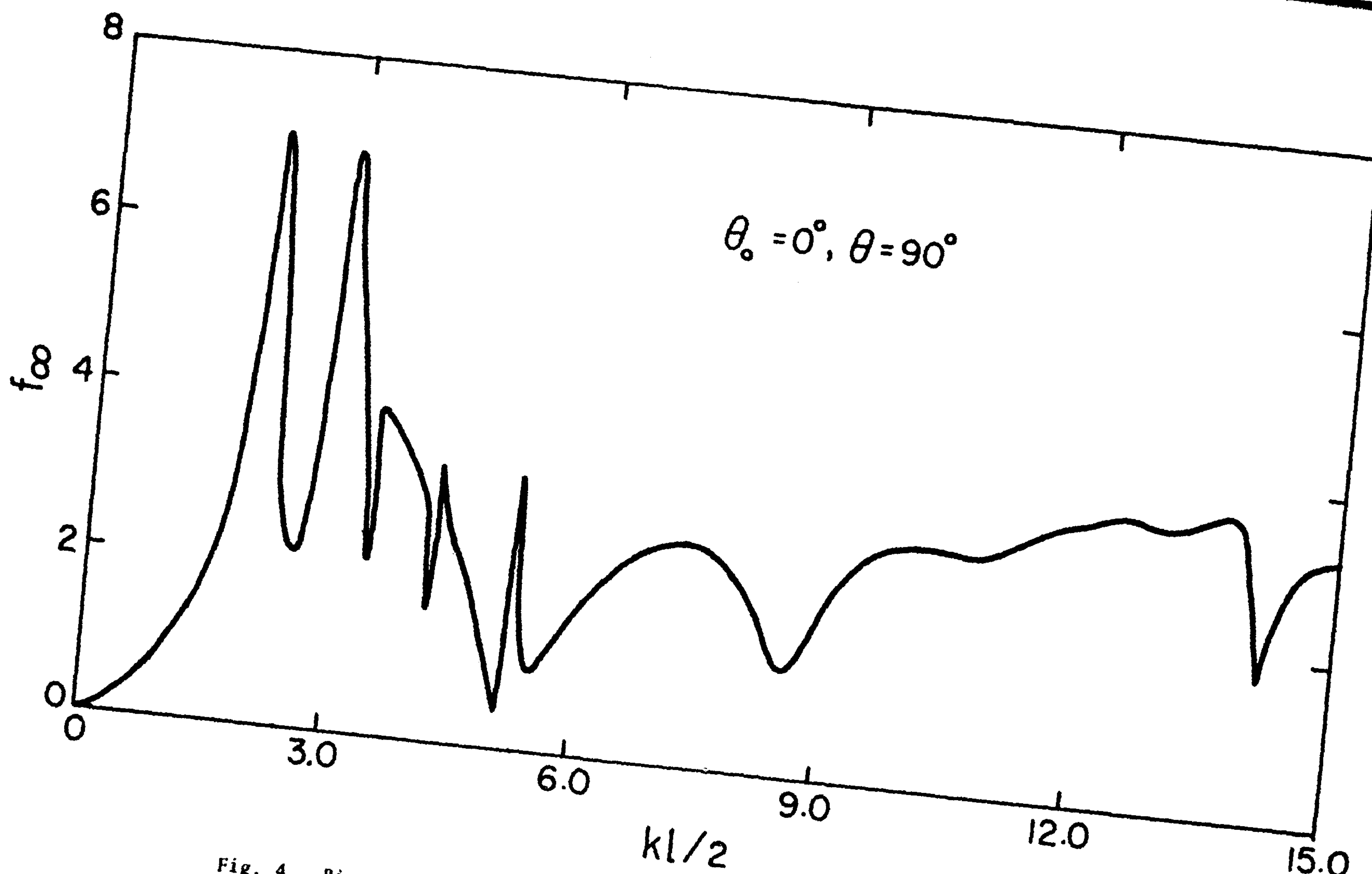

Fig. 4. Bistatic form function as a function of non-dimensional wavenumber (θ_o=0°, θ=90°).

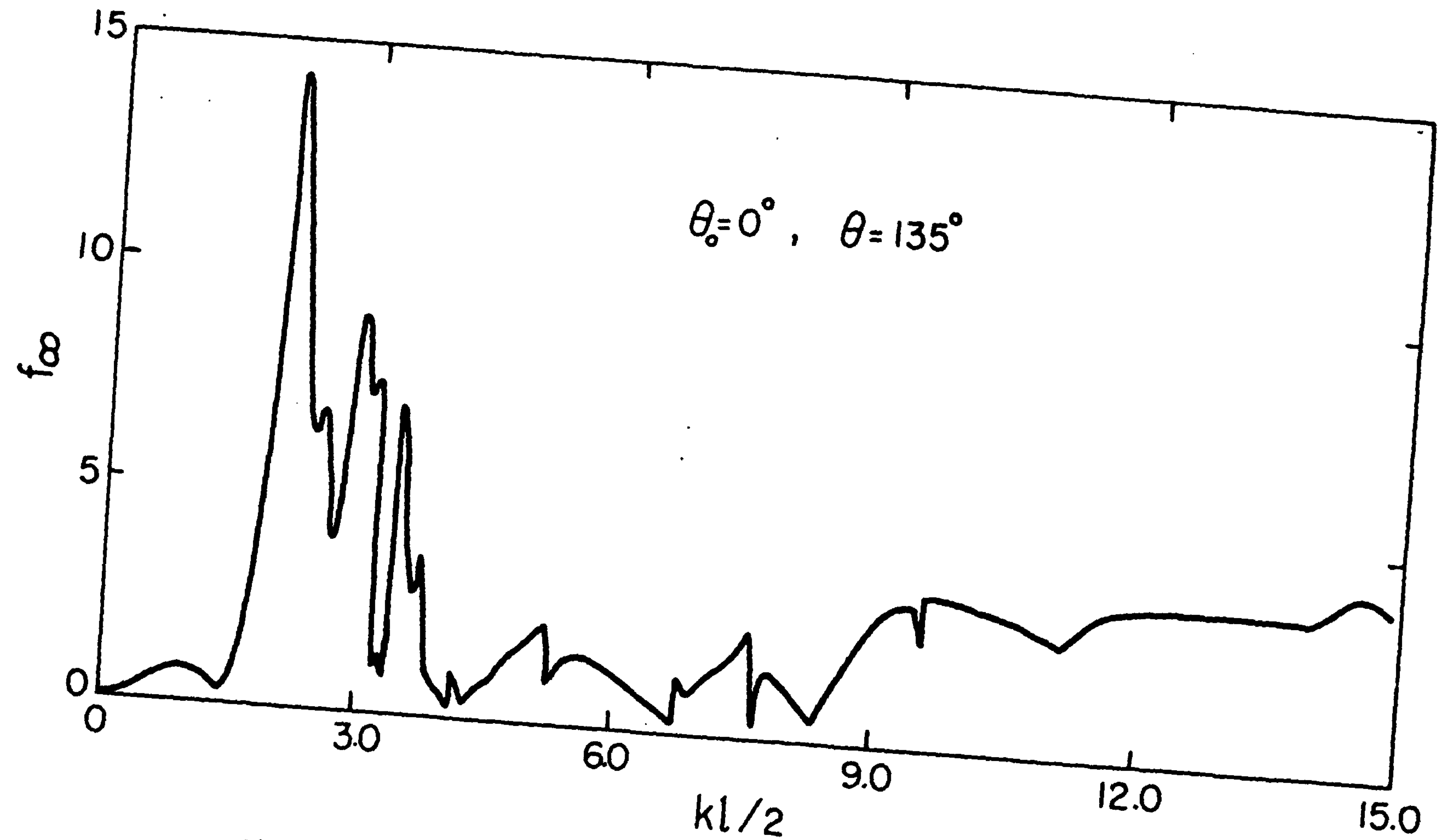

Fig. 5. Bistatic form function as function of non-dimensional wavenumber (θ_o=0°, θ=135°).

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